MECHANICS.

205. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Given two points A and B not in the same horizontal nor in the same vertical line; to find the path from A to B along which a particle will slide from rest under the force of gravity alone so that the average velocity along the curve shall be a maximum.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let the particle start from rest at the point (x_1, y_1), where the axis of x is vertical and the axis of y horizontal. Then

$$v=\frac{ds}{dt}=\sqrt{[2g(x-x_1)]}=\frac{dx}{dt}\sqrt{1+\left(\frac{dy}{dx}\right)^2}.$$

Since v=a maximum, t=a minimum.

$$\therefore t=U=\int_{x_1}^{x_2}\frac{\sqrt{[1+(dy/dx)^2]}\,dx}{\sqrt{[2g(x-x_1)]}}=\text{minimum}.$$

By Calculus of Variations, since the expression in the right hand member does not contain y explicitly, the differential of this expression is equal to a constant.

$$\therefore \frac{dy/dx}{\sqrt{[2g(x-x_1)]}\sqrt{[1+(dy/dx)^2]}}=C.$$

$$\therefore \left(\frac{dy}{dx}\right)^2=C^2\left[1+\left(\frac{dy}{dx}\right)^2\right][2g(x-x_1)].$$

$$\therefore \frac{dy}{dx}=\pm\frac{C\sqrt{[2g(x-x_1)]}}{\sqrt{[1-2gC^2(x-x_1)]}},$$

the differential equation of a cycloid. This is Bernoulli's famous problem and is solved in almost every work on the Calculus of Variations as well as Dynamics.

206. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

A rigid square $ABDC$ made by smooth wires is fixed with A vertically above D. Two small equal spherical elastic beads slide down BD, CD, starting simultaneously from B and C. Find the ratio of their velocities of approach and separation at D, and how far they will separate after impact.